Quadratic Equations

Question 1.
Find which of the following equations are quadratic:

Solution 1(i)
$(3x - 1)^2 = 5(x + 8)$
$\Rightarrow (9x^2 - 6x + 1) = 5x + 40$
$\Rightarrow 9x^2 - 11x - 39 = 0$; which is of the form $ax^2 + bx + c = 0$.
$\therefore$ Given equation is a quadratic equation.

Solution 1(ii)
$5x^2 - 8x = -3(7 - 2x)$
$\Rightarrow 5x^2 - 8x = 6x - 21$
$\Rightarrow 5x^2 - 14x + 21 = 0$; which is of the form $ax^2 + bx + c = 0$.
$\therefore$ Given equation is a quadratic equation.

Solution 1(iii)
$(x - 4)(3x + 1) = (3x - 1)(x + 2)$
$\Rightarrow 3x^2 + x - 12x - 4 = 3x^2 + 6x - x - 2$
$\Rightarrow 16x + 2 = 0$; which is not of the form $ax^2 + bx + c = 0$.
$\therefore$ Given equation is not a quadratic equation.

Solution 1(iv)
$x^2 + 5x - 5 = (x - 3)^2$
$\Rightarrow x^2 + 5x - 5 = x^2 - 6x + 9$
$\Rightarrow 11x - 14 = 0$; which is not of the form $ax^2 + bx + c = 0$.
$\therefore$ Given equation is not a quadratic equation.

Solution 1(v)
$7x^3 - 2x^2 + 10 = (2x - 5)^2$
$\Rightarrow 7x^3 - 2x^2 + 10 = 4x^2 - 20x + 25$
$\Rightarrow 7x^3 - 6x^2 + 20x - 15 = 0$; which is not of the form $ax^2 + bx + c = 0$.
$\therefore$ Given equation is not a quadratic equation.

Solution 1(vi)
$(x - 1)^2 + (x + 2)^2 + 3(x + 1) = 0$
$\Rightarrow x^2 - 2x + 1 + x^2 + 4x + 4 + 3x + 3 = 0$
$\Rightarrow 2x^2 + 5x + 8 = 0$; which is of the form $ax^2 + bx + c = 0$.
$\therefore$ Given equation is a quadratic equation.

Question 2(i)
Is x = 5 a solution of the quadratic equation $x^2 - 2x - 15 = 0$?

Solution:
$x^2 - 2x - 15 = 0$
For x = 5 to be solution of the given quadratic equation it should satisfy the equation.
So, substituting x = 5 in the given equation, we get
L.H.S = $(5)^2 - 2(5) - 15$
= 25 – 10 – 15
= 0
= R.H.S
Hence, x = 5 is a solution of the quadratic equation $x^2 - 2x - 15 = 0$.

Question 2(ii).
Is x = -3 a solution of the quadratic equation $2x^2 - 7x + 9 = 0$?

Solution:
$2x^2 - 7x + 9 = 0$
For x = -3 to be solution of the given quadratic equation it should satisfy the equation
So, substituting x = 5 in the given equation, we get
L.H.S =$2(-3)^2 - 7(-3) + 9$
= 18 + 21 + 9
= 48
≠ R.H.S
Hence, x = -3 is not a solution of the quadratic equation $2x^2 - 7x + 9 = 0$.

Question 3.
If $\sqrt{\frac{2}{3}}$ is a solution of equation $3x^2 + mx + 2 = 0$, find the value of m.

Solution:
For x = $\sqrt{\frac{2}{3}}$ to be solution of the given quadratic equation it should satisfy the equation
So, substituting x = $\sqrt{\frac{2}{3}}$ in the given equation, we get

$$3\left(\sqrt{\frac{2}{3}}\right)^2 + m\left(\sqrt{\frac{2}{3}}\right) + 2 = 0$$

$$\Rightarrow 3\left(\frac{2}{3}\right) + m\left(\sqrt{\frac{2}{3}}\right) + 2 = 0$$

$$\Rightarrow m = -4 \times \sqrt{\frac{3}{2}} = -2\sqrt{6}$$

$$\therefore m = -2\sqrt{6}$$

Question 4.
$\frac{2}{3}$ and 1 are the solutions of equation $mx^2 + nx + 6 = 0$. Find the values of m and n.

Solution:

For x = $\frac{2}{3}$ and x = 1 to be solutions of the given quadratic equation it should satisfy the equation

So, substituting x = $\frac{2}{3}$ and x = 1 in the given equation, we get

$$\begin{array}{l|l} m\left(\frac{2}{3}\right)^2 + n\left(\frac{2}{3}\right) + 6 = 0 & m(1)^2 + n(1) + 6 = 0 \\ \Rightarrow m\left(\frac{4}{9}\right) + n\left(\frac{2}{3}\right) + 6 = 0 & \Rightarrow m + n + 6 = 0 \\ \Rightarrow 4m + 6n + 54 = 0 \ldots (1) & \Rightarrow m + n + 6 = 0 \ldots (2) \end{array}$$

Solving equations (1) and (2) simultaneously,
4m + 6n + 54 = 0 .....(1)
m + n + 6 = 0 ....(2)
(1) – (2) × 6
⇒ -2m + 18 = 0
⇒ m = 9
Substitute in (2)
⇒ n = -15

Question 5.
If 3 and -3 are the solutions of equation $ax^2 + bx - 9 = 0$. Find the values of a and b.

Solution:

For x = 3 and x = -3 to be solutions of the given quadratic equation it should satisfy the equation

So, substituting x = 3 and x = -3 in the given equation, we get

$$\begin{array}{l|l} a(3)^2 + b(3) - 9 = 0 & a(-3)^2 + b(-3) - 9 = 0 \\ \Rightarrow a(9) + b(3) - 9 = 0 & \Rightarrow a(9) - b(3) - 9 = 0 \\ \Rightarrow 9a + 3b - 9 = 0 \ldots (1) & \Rightarrow 9a - 3b - 9 = 0 \ldots (2) \end{array}$$

Solving equations (1) and (2) simultaneously,

$9a + 3b - 9 = 0$ ...(1)
$9a - 3b - 9 = 0$ ...(2)
(1) + (2)
$\Rightarrow 18a - 18 = 0$
$\Rightarrow a = 1$
Substitute in (2)
$\Rightarrow b = 0$

Exercise 5B

Question 1.
Without solving, comment upon the nature of roots of each of the following equations :
(i) $7x^2 - 9x + 2 = 0$
(ii) $6x^2 - 13x + 4 = 0$
(iii) $25x^2 - 10x + 1 = 0$
(iv) $x^2 + 2\sqrt{3}x - 9 = 0$
(v) $x^2 - ax - b^2 = 0$
(vi) $2x^2 + 8x + 9 = 0$

Solution:

(i) $7x^2 - 9x + 2 = 0$
$a = 7,\ b = -9$ and $c = 2$
$\therefore$ Discriminant $= b^2 - 4ac$
$= (-9)^2 - 4(7)(2)$
$= 81 - 56 = 25$
Since $D > 0$, then equation has two real and unequal roots.

(ii) $6x^2 - 13x + 4 = 0$
$a = 6,\ b = -13$ and $c = 4$
$\therefore$ Discriminant $= b^2 - 4ac$
$= (-13)^2 - 4(6)(4)$
$= 169 - 96 = 73$
Since 73 is not a perfect square, roots are irrational
Since $D > 0$, then equation has two irrational and unequal roots.

(iii) $25x^2 - 10x + 1 = 0$
$a = 25,\ b = -10$ and $c = 1$
$\therefore$ Discriminant $= b^2 - 4ac$

$=(-10)^2 - 4(25)(1)$

$=100 - 100 = 0$

Since D=0, then equation has two real and equal roots.

(iv) $x^2 + 2\sqrt{3}x - 9 = 0$

$a = 1,\ b=2\sqrt{3}$ and $c = -9$

$\therefore$ Discriminant$=b^2 - 4ac$

$=(2\sqrt{3})^2 - 4(1)(-9)$

$=12+36 = 48$

Since 48 is not a perfect square, roots are irrational

Since D>0, then equation has two irrational and unequal roots.

(v) $x^2 - ax - b^2 = 0$

$A = 1,\ B = -a$ and $C = -b^2$

$\therefore$ Discriminant$=B^2 - 4AC$

$=(-a)^2 - 4(1)(-b^2)$

$=a^2+4b^2$ = a positive value

Since a^2+4b^2 is not a perfect square, roots are irrational.

Since D>0, then equation has two irrational and unequal roots.

(vi) $2x^2 + 8x + 9 = 0$

$a = 2,\ b=8$ and $c = 9$

$\therefore$ Discriminant$=b^2 - 4ac$

$=(8)^2 - 4(2)(9)$

$=64 - 72 = -18$ = a negative value

Since D<0, then equation has no real roots.

Question 2.

Find the value of p, if the following quadratic equation has equal roots : $4x^2 - (p - 2)x + 1 = 0$

Solution:

$4x^2 - (p-2)x + 1 = 0$

Here $a = 4$, $b = -(p-2)$ and $c = 1$

Given: equation has equal roots

Then $D = 0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow [-(p-2)]^2 - 4(4)(1) = 0$

$\Rightarrow p^2 + 4 - 4p - 16 = 0$

$\Rightarrow p^2 - 4p - 12 = 0$

$\Rightarrow p^2 - 6p + 2p - 12 = 0$

$\Rightarrow p(p-6) + 2(p-6) = 0$

$\Rightarrow (p-6)(p+2) = 0$

then $p - 6 = 0$ or $p + 2 = 0$

$\Rightarrow$ $p = 6$ or $p = -2$

Question 3.

Find the value of 'p', if the following quadratic equations have equal roots : $x^2 + (p - 3)x + p = 0$

Solution:

$x^2 + (p - 3)x + p = 0$

Here, $a = 1$, $b = (p - 3)$, $c = p$

Since, the roots are equal,

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (p - 3)^2 - 4(1)(p) = 0$

$\Rightarrow p^2 + 9 - 6p - 4p = 0$

$\Rightarrow p^2 - 10p + 9 = 0$

$\Rightarrow p^2 - 9p - p + 9 = 0$

$\Rightarrow p(p - 9) - 1(p - 9) = 0$

$\Rightarrow (p - 9)(p - 1) = 0$

$\Rightarrow p - 9 = 0$ or $p - 1 = 0$

$\Rightarrow p = 9$ or $p = 1$

Question 4.

The equation $3x^2 - 12x + (n - 5) = 0$ has equal roots. Find the value of n.

Solution:

$3x^2 - 12x + (n - 5) = 0$
Here $a = 3$, $b = -12$ and $c = n - 5$
Given: equation has equal roots
Then $D = 0$
$\Rightarrow b^2 - 4ac = 0$
$\Rightarrow [-12]^2 - 4(3)(n - 5) = 0$
$\Rightarrow 144 - 12n + 60 = 0$
$\Rightarrow -12n = -204$
$\Rightarrow n = \frac{-204}{-12} = 17$

Question 5.
Find the value of m, if the following equation has equal roots : $(m - 2)x^2 - (5+m)x + 16 = 0$

Solution:

$(m - 2)x^2 - (5 + m)x + 16 = 0$
Here $a = m - 2$, $b = -(5 + m)$ and $c = 16$
Given: equation has equal roots
Then $D = 0$
$\Rightarrow b^2 - 4ac = 0$
$\Rightarrow [-(5 + m)]^2 - 4(m - 2)(16) = 0$
$\Rightarrow 25 + m^2 + 10m - 64m + 128 = 0$
$\Rightarrow m^2 - 54m + 153 = 0$
$\Rightarrow m^2 - 51m - 3m + 153 = 0$
$\Rightarrow m(m - 51) - 3(m - 51) = 0$
$\Rightarrow (m - 51)(m - 3) = 0$
then $m - 51 = 0$ or $m - 3 = 0$
$\Rightarrow$ $m = 51$ or $m = 3$

Question 6.
Find the value of p for which the equation $3x^2 - 6x + k = 0$ has distinct and real roots.

Solution:

$3x^2 - 6x + k = 0$
Here, $a = 3$, $b = -6$ and $c = k$
Since the roots are distinct and real,
$b^2 - 4ac > 0$
$\Rightarrow (-6)^2 - 4 \times 3 \times k > 0$
$\Rightarrow 36 - 12k > 0$
$\Rightarrow 36 > 12k$
$\Rightarrow 3 > k$
$\Rightarrow k < 3$

Exercise 5C

Question 1.
Solve : $x^2 - 10x - 24 = 0$

Solution:

$x^2 - 10x - 24 = 0$
$\Rightarrow x^2 - 12x + 2x - 24 = 0$
$\Rightarrow x(x - 12) + 2(x - 12) = 0$
$\Rightarrow (x - 12)(x + 2) = 0$
since $x-12=0$ or $x+2=0$
then $x=12$ or $x= -2$

Question 2.
Solve : $x^2 - 16 = 0$

Solution:

$x^2 - 16 = 0$
$\Rightarrow x^2 - 4^2 = 0$
$\Rightarrow (x + 4)(x - 4) = 0$
Since $x+4=0$ or $x-4=0$
then $x= -4$ or $x=4$

Question 3.

Solve : $2x^2 - \frac{1}{2}x = 0$

Solution:

$2x^2 - \frac{1}{2}x = 0$

$\Rightarrow \quad x(2x-\frac{1}{2}) = 0$

since $x=0$ or $2x-\frac{1}{2} = 0$

then $x=0$ or $x=\frac{1}{4}$

Question 4.

Solve : $x(x - 5) = 24$

Solution:

$x(x-5)=24$

$\Rightarrow \quad x^2 - 5x - 24 = 0$

$\Rightarrow \quad x^2 - 8x + 3x - 24 = 0$

$\Rightarrow \quad x(x-8)+3(x-8)=0$

$\Rightarrow \quad (x-8)(x+3)=0$

Since $x-8=0$ or $x+3=0$

then $x=8$ or $x= -3$

Question 5.

Solve : $\frac{9}{2}x = 5 + x^2$

Solution:

$\frac{9}{2}x = 5 + x^2$

$\Rightarrow \quad 9x = 10+2x^2$

$\Rightarrow \quad 2x^2 - 9x + 10 = 0$

$\Rightarrow 2x^2 - 5x - 4x + 10 = 0$

$\Rightarrow x(2x-5)-2(2x-5)=0$

$\Rightarrow (2x-5)(x-2)=0$

Since $2x-5=0$ or $x-2=0$

then $x=\frac{5}{2}$ or $x=2$

Question 6.

Solve: $\frac{6}{x} = 1 + x$

Solution:

$\frac{6}{x} = 1 + x$

$\Rightarrow 6 = x + x^2$

$\Rightarrow x^2 + x - 6 = 0$

$\Rightarrow x^2 + 3x - 2x - 6 = 0$

$\Rightarrow x(x+3)-2(x+3)=0$

$\Rightarrow (x+3)(x-2)=0$

since $x+3=0$ or $x-2=0$

then $x=-3$ or $x=2$

Question 7.

Solve: $x=\frac{3x+1}{4x}$

Solution:

$x=\frac{3x+1}{4x}$

$\Rightarrow 4x^2 = 3x + 1$

$\Rightarrow 4x^2 - 3x - 1 = 0$

$\Rightarrow 4x^2 - 4x + x - 1 = 0$

$\Rightarrow 4x(x-1)+1(x-1)=0$

$\Rightarrow (x-1)(4x+1)=0$

Since $x-1=0$ or $4x+1=0$

then $x=1$ or $x=\frac{-1}{4}$

Question 8.

Solve : $x+\frac{1}{x}=2.5$

Solution:

$x+\frac{1}{x}=2.5$

$\Rightarrow \frac{x^2+1}{x}=\frac{5}{2}$

$\Rightarrow 2x^2+2=5x$

$\Rightarrow 2x^2-5x+2=0$

$\Rightarrow 2x^2-4x-x+2=0$

$\Rightarrow 2x(x-2)-1(x-2)=0$

$\Rightarrow (x-2)(2x-1)=0$

Since $x-2=0$ or $2x-1=0$

then $x=2$ or $x=\frac{1}{2}$

Question 9.

Solve : $(2x-3)^2=49$

Solution:

$(2x-3)^2=49$

Taking square root on both sides

$2x-3=\pm 7$

When $2x-3=7 \Rightarrow 2x=10 \Rightarrow x=5$

and, when $2x-3=-7 \Rightarrow 2x=-4 \Rightarrow x=-2$

Question 10.
Solve : $2(x^2 - 6) = 3(x - 4)$

Solution:

$$2(x^2 - 6) = 3(x - 4)$$
$$\Rightarrow 2x^2 - 12 = 3x - 12$$
$$\Rightarrow 2x^2 - 3x = 0$$
$$\Rightarrow x(2x - 3) = 0$$
since $x=0$ or $2x-3=0$
then $x=0$ or $x=\frac{3}{2}$

Question 11.
Solve : $(x + 1)(2x + 8) = (x + 7)(x + 3)$

Solution:

$$(x+1)(2x+8)=(x+7)(x+3)$$
$$\Rightarrow 2x^2 + 8x + 2x + 8 = x^2 + 3x + 7x + 21$$
$$\Rightarrow 2x^2 + 10x + 8 = x^2 + 10x + 21$$
$$\Rightarrow x^2 - 13 = 0$$
$$\Rightarrow x^2 - \left(\sqrt{13}\right)^2 = 0$$
$$\Rightarrow (x + \sqrt{13})(x - \sqrt{13}) = 0$$
If $x+\sqrt{13} = 0$ or $x - \sqrt{13} = 0$
$\Rightarrow x = -\sqrt{13}$ or $x=\sqrt{13}$

Question 12.

Solve : $x^2 - (a + b)x + ab = 0$

Solution:

$x^2 - (a+b)x + ab = 0$

$\Rightarrow x^2 - ax - bx + ab = 0$

$\Rightarrow x(x-a) - b(x-a) = 0$

$\Rightarrow (x-a)(x-b) = 0$

since $x - a = 0$ or $x - b = 0$

then $x = a$ or $x = b$

Question 13.

$(x + 3)^2 - 4(x + 3) - 5 = 0$

Solution:

$(x+3)^2 - 4(x+3) - 5 = 0$

Let $x+3=y$

then $y^2 - 4y - 5 = 0$

$\Rightarrow y^2 - 5y + y - 5 = 0$

$\Rightarrow y(y-5)+1(y-5)=0$

$\Rightarrow (y-5)(y+1)=0$

If $y-5=0$ or $y+1=0$

then $y=5$ or $y=-1$

$\Rightarrow x+3=5$ or $x+3=-1$

$\Rightarrow x=2$ or $x=-4$

Question 14.

$4(2x - 3)^2 - (2x - 3) - 14 = 0$

Solution:

$4(2x-3)^2 - (2x-3) - 14 = 0$

Let $2x-3=y$

then $4y^2 - y - 14 = 0$

$\Rightarrow 4y^2 - 8y + 7y - 14 = 0$

$\Rightarrow 4y(y-2) + 7(y-2) = 0$

$$\Rightarrow (y-2)(4y+7)=0$$

If $y-2=0$ or $4y+7=0$

$\Rightarrow y=2$ or $y=\frac{-7}{4}$

$\Rightarrow 2x-3=2$ or $2x-3=\frac{-7}{4}$

$\Rightarrow 2x=5$ or $2x=\frac{5}{4}$

$\Rightarrow x=\frac{5}{2}$ or $x=\frac{5}{8}$

Question 15.

Solve: $\frac{3x-2}{2x-3}=\frac{3x-8}{x+4}$

Solution:

$$\frac{3x-2}{2x-3}=\frac{3x-8}{x+4}$$

$$\Rightarrow (3x-2)(x+4)=(2x-3)(3x-8)$$

$$\Rightarrow 3x^2+12x-2x-8=6x^2-16x-9x+24$$

$$\Rightarrow 3x^2+10x-8=6x^2-25x+24$$

$$\Rightarrow 3x^2-35x+32=0$$

$$\Rightarrow 3x^2-32x-3x+32=0$$

$$\Rightarrow x(3x-32)-1(3x-32)=0$$

$$\Rightarrow (x-1)(3x-32)=0$$

If $x-1=0$ or $3x-32=0$

$\Rightarrow x=1$ or $x=\frac{32}{3}=10\frac{2}{3}$

Question 16.

$2x^2 - 9x + 10 = 0$, When

(i) $x \in N$

(ii) $x \in Q$

Solution:

$2x^2 - 9x + 10 = 0$

$\Rightarrow 2x^2 - 5x - 4x + 10 = 0$

$\Rightarrow x(2x - 5) - 2(2x - 5) = 0$

$\Rightarrow (2x - 5)(x - 2) = 0$

$\Rightarrow 2x - 5 = 0$ or $x - 2 = 0$

$\Rightarrow x = \frac{5}{2}$ or $x = 2$

(i) When $x \in N$, we have $x = 2$

(ii) When $x \in Q$, we have $x = 2, \frac{5}{2}$

Question 17.

Solve: $\frac{x-3}{x+3} + \frac{x+3}{x-3} = 2\frac{1}{2}$

Solution:

$\frac{x-3}{x+3} + \frac{x+3}{x-3} = 2\frac{1}{2}$

$\Rightarrow \frac{(x-3)^2 + (x+3)^2}{(x+3)(x-3)} = \frac{5}{2}$

$\Rightarrow \frac{x^2 - 6x + 9 + x^2 + 6x + 9}{x^2 - 9} = \frac{5}{2}$

$\Rightarrow 2(2x^2 + 18) = 5(x^2 - 9)$

$\Rightarrow 4x^2 + 36 = 5x^2 - 45$

$\Rightarrow x^2 - 81 = 0$

$\Rightarrow x^2 - 9^2 = 0$

$\Rightarrow (x + 9)(x - 9) = 0$

If $x + 9 = 0$ or $x - 9 = 0$

then $x = -9$ or $x = 9$

Question 18.

Solve: $\frac{4}{x+2}-\frac{1}{x+3}=\frac{4}{2x+1}$

Solution:

$$\frac{4}{x+2}-\frac{1}{x+3}=\frac{4}{2x+1}$$
$$\Rightarrow \frac{4(x+3)-1(x+2)}{(x+2)(x+3)}=\frac{4}{2x+1}$$
$$\Rightarrow \frac{4x+12-x-2}{x^2+2x+3x+6}=\frac{4}{2x+1}$$
$$\Rightarrow \frac{3x+10}{x^2+5x+6}=\frac{4}{2x+1}$$
$$\Rightarrow (3x+10)(2x+1)=4(x^2+5x+6)$$
$$\Rightarrow 6x^2+3x+20x+10=4x^2+20x+24$$
$$\Rightarrow 2x^2+3x-14=0$$
$$\Rightarrow 2x^2+7x-4x-14=0$$
$$\Rightarrow x(2x+7)-2(2x+7)=0$$
$$\Rightarrow (2x+7)(x-2)=0$$

If $2x+7=0$ or $x-2=0$

then $x=\frac{-7}{2}$ or $x=2$

Question 19.

Solve: $\frac{5}{x-2}-\frac{3}{x+6}=\frac{4}{x}$

Solution:

$$\frac{5}{x-2}-\frac{3}{x+6}=\frac{4}{x}$$
$$\Rightarrow \frac{5(x+6)-3(x-2)}{(x-2)(x+6)}=\frac{4}{x}$$
$$\Rightarrow \frac{5x+30-3x+6}{x^2+6x-2x-12}=\frac{4}{x}$$

$$\Rightarrow \frac{2x+36}{x^2+4x-12} = \frac{4}{x}$$
$$\Rightarrow 4x^2 + 16x - 48 = 2x^2 + 36x$$
$$\Rightarrow 2x^2 - 20x - 48 = 0$$
$$\Rightarrow x^2 - 10x - 24 = 0$$
$$\Rightarrow x^2 - 12x + 2x - 24 = 0$$
$$\Rightarrow x(x-12) + 2(x-12) = 0$$
$$\Rightarrow (x-12)(x+2) = 0$$

If $x - 12 = 0$ or $x+2=0$

then $x=12$ or $x=-2$

Question 20.

Solve : $\left(1+\frac{1}{x+1}\right)\left(1-\frac{1}{x-1}\right) = \frac{7}{8}$

Solution:

$$\left(1+\frac{1}{x+1}\right)\left(1-\frac{1}{x-1}\right) = \frac{7}{8}$$
$$\Rightarrow \left(\frac{x+1+1}{x+1}\right)\left(\frac{x-1-1}{x-1}\right) = \frac{7}{8}$$
$$\Rightarrow \left(\frac{x+2}{x+1}\right)\left(\frac{x-2}{x-1}\right) = \frac{7}{8}$$
$$\Rightarrow \frac{x^2-4}{x^2-1} = \frac{7}{8}$$
$$\Rightarrow 8x^2 - 32 = 7x^2 - 7$$
$$\Rightarrow x^2 = 25$$
$$\Rightarrow x = \pm 5$$

Question 21.

Find the quadratic equation, whose solution set is :

(i) {3, 5} (ii) {-2, 3}

Solution:

(i)Since solution set is $\{3,5\}$

$\Rightarrow x=3 \quad \text{or} \quad x=5$

$\Rightarrow x-3=0 \quad \text{or} \quad x-5=0$

$\Rightarrow (x-3)(x-5)=0$

$\Rightarrow x^2-5x-3x+15=0$

$\Rightarrow x^2-8x+15=0$ which is the required equation.

(ii)Since solution set is $\{-2,3\}$

$\Rightarrow x=-2 \quad \text{or} \quad x=3$

$\Rightarrow x+2=0 \quad \text{or} \quad x-3=0$

$\Rightarrow (x+2)(x-3)=0$

$\Rightarrow x^2-3x+2x-6=0$

$\Rightarrow x^2-x-6=0$ which is the required equation.

(iii)Since solution set is $\{5,-4\}$

$\Rightarrow x=5 \quad \text{or} \quad x=-4$

$\Rightarrow x-5=0 \quad \text{or} \quad x+4=0$

$\Rightarrow (x-5)(x+4)=0$

$\Rightarrow x^2-5x+4x-20=0$

$\Rightarrow x^2-x-20=0$ which is the required equation.

(iv)Since solution set is $\left\{-3,\frac{-2}{5}\right\}$

$\Rightarrow x=-3 \quad \text{or} \quad x=\frac{-2}{5}$

$\Rightarrow x+3=0 \quad \text{or} \quad 5x+2=0$

$\Rightarrow (x+3)(5x+2)=0$

$\Rightarrow 5x^2+2x+15x+6=0$

$\Rightarrow 5x^2+17x+6=0$ which is the required equation.

Question 22.

Solve : $\frac{x}{3}+\frac{3}{6-x}=\frac{2(6+x)}{15};(x\neq 6)$

Solution:

$$\frac{x}{3}+\frac{3}{6-x}=\frac{2(6+x)}{15}$$
$$\Rightarrow \frac{x(6-x)+3\times 3}{3(6-x)}=\frac{12+2x}{15}$$
$$\Rightarrow \frac{x(6-x)+3\times 3}{6-x}=\frac{12+2x}{5}$$
$$\Rightarrow \frac{6x-x^2+9}{6-x}=\frac{12+2x}{5}$$
$$\Rightarrow 30x-5x^2+45=72+12x-12x-2x^2$$
$$\Rightarrow 30x-5x^2+45=72-2x^2$$
$$\Rightarrow 3x^2-30x+27=0$$
$$\Rightarrow x^2-10x+9=0$$
$$\Rightarrow x^2-9x-x+9=0$$
$$\Rightarrow x(x-9)-1(x-9)=0$$
$$\Rightarrow (x-9)(x-1)=0$$
$$\Rightarrow x-9=0 \text{ or } x-1=0$$
$$\Rightarrow x=9 \text{ or } x=1$$

Question 23.

Solve the equation $9x^2+\frac{3x}{4}+2=0$, if possible, for real values of x.

Solution:

$$9x^2+\frac{3x}{4}+2=0$$
$$\Rightarrow \frac{36x^2+3x+8}{4}=0$$
$$\Rightarrow 36x^2+3x+8=0$$

Here, $a=36$, $b=3$ and $c=8$

$$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{(3)^2 - 4 \times 36 \times 8}}{2 \times 36}$$

$$= \frac{-3 \pm \sqrt{9 - 1152}}{72}$$

$$= \frac{-3 \pm \sqrt{-1143}}{72}$$

Since $\sqrt{-1143}$ is not possible, we cannot solve the given equation for x.

Question 24.
Find the value of x, if a + 1=0 and x^2 + ax − 6 =0.

Solution:

If a+1=0, then a = -1
Put this value in the given equation $x^2 + ax - 6 = 0$

$$x^2 - x - 6 = 0$$

$$\Rightarrow x^2 - 3x + 2x - 6 = 0$$

$$\Rightarrow x(x-3) + 2(x-3) = 0$$

$$\Rightarrow (x-3)(x+2) = 0$$

If $x - 3 = 0$ or $x+2=0$
then $x=3$ or $x=-2$

Question 25.
Find the value of x, if a + 7=0; b + 10=0 and $12x^2$ = ax − b.

Solution:
If a + 7 =0, then a = -7
and b + 10 =0, then b = − 10
Put these values of a and b in the given equation

$$12x^2 = (-7)x - (-10)$$

$$\Rightarrow 12x^2 + 7x - 10 = 0$$

$$\Rightarrow 12x^2 + 15x - 8x - 10 = 0$$

$$\Rightarrow 3x(4x+5) - 2(4x+5) = 0$$

$\Rightarrow (4x+5)(3x-2)=0$

If $4x+5=0$ or $3x-2=0$

then $x=\frac{-5}{4}$ or $x=\frac{2}{3}$

Question 26.

Use the substitution y= 2x +3 to solve for x, if $4(2x+3)^2 - (2x+3) - 14 = 0$.

Solution:

$4(2x+3)^2-(2x+3)-14=0$

Put $2x+3=y$

$4y^2-y-14=0$

$\Rightarrow 4y^2-8y+7y-14=0$

$\Rightarrow 4y(y-2)+7(y-2)=0$

$\Rightarrow (y-2)(4y+7)=0$

If $y-2=0$ or $4y+7=0$

then $2x+3-2=0$ or $4(2x+3)+7=0$

$\Rightarrow 2x=-1$ or $8x=-19$

$\Rightarrow x=\frac{-1}{2}$ or $x=\frac{-19}{8}$

Question 27.

Without solving the quadratic equation $6x^2 - x - 2=0$, find whether x = 2/3 is a solution of this equation or not.

Solution:

Consider the equation, $6x^2-x-2=0$

Put $x=\frac{2}{3}$ in L.H.S.

$$\text{L.H.S.} = 6\left(\frac{2}{3}\right)^2-\left(\frac{2}{3}\right)-2$$

$$=\frac{24}{9}-\frac{2}{3}-2$$

$$= \frac{24 - 6 - 18}{9} = 0 = \text{R.H.S.}$$

Since L.H.S.= R.H.S., then $x = \frac{2}{3}$ is a solution of the given equation.

Question 28.
Determine whether x = -1 is a root of the equation $x^2 - 3x + 2 = 0$ or not.

Solution:
$x^2 - 3x + 2 = 0$
Put x = -1 in L.H.S.
L.H.S. = $(-1)^2 - 3(-1) + 2$
$= 1 + 3 + 2 = 6 \neq$ R.H.S
Then x = -1 is not the solution of the given equation.

Question 29.
If x = 2/3 is a solution of the quadratic equation $7x^2 + mx - 3 = 0$; Find the value of m.

Solution:

$7x^2 + mx - 3 = 0$

Given $x = \frac{2}{3}$ is the solution of the given equation.

Put given value of x in the given equation

$$7\left(\frac{2}{3}\right)^2 + m\left(\frac{2}{3}\right) - 3 = 0$$

$$\Rightarrow \frac{28}{9} + \frac{2m}{3} - 3 = 0$$

$$\Rightarrow 28 + 6m - 27 = 0$$

$$\Rightarrow 6m = -1$$

$$\Rightarrow m = \frac{-1}{6}$$

Question 30.
If x = -3 and x = 2/3 are solutions of quadratic equation $mx^2 + 7x + n = 0$, find the values of m and n.

Solution:

$mx^2 + 7x + n = 0$

Put $x = -3$ in given equation

$m(-3)^2 + 7(-3) + n = 0$

$\Rightarrow 9m - 21 + n = 0$

$\Rightarrow 9m + n = 21 - - - - - (1)$

Put $x = \frac{2}{3}$ in given equation

$m\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + n = 0$

$\Rightarrow \frac{4m}{9} + \frac{14}{3} + n = 0$

$\Rightarrow 4m + 9n = -42 - - - - - (2)$

solving these equations we get

$m = 3$ and $n = -6$

Question 31.

If quadratic equation $x^2 - (m + 1)x + 6 = 0$ has one root as $x = 3$; find the value of m and the root of the equation.

Solution:

$x^2 - (m + 1)x + 6 = 0$

Put $x = 3$ in the given equation

$(3)^2 - (m + 1)(3) + 6 = 0$

$\Rightarrow 9 - 3m - 3 + 6 = 0$

$\Rightarrow -3m = -12$

$\Rightarrow m = 4$

Put this value of m in the given equation, we get

$x^2 - 5x + 6 = 0$

$\Rightarrow x^2 - 3x - 2x + 6 = 0$

$\Rightarrow x(x - 3) - 2(x - 3) = 0$

$\Rightarrow (x - 3)(x - 2) = 0$

If $x - 3 = 0$ or $x - 2 = 0$

then $x = 3$ or $x = 2$

$\therefore$ 2 is the other root of the given equation.

Question 32.
Given that 2 is a root of the equation $3x^2 - p(x + 1) = 0$ and that the equation $px^2 - qx + 9 = 0$ has equal roots, find the values of p and q.

Solution:

Since 2 is a root of the equation $3x^2 - p(x + 1) = 0$

$\Rightarrow 3(2)^2 - p(2 + 1) = 0$

$\Rightarrow 3 \times 4 - 3p = 0$

$\Rightarrow 12 - 3p = 0$

$\Rightarrow 3p = 12$

$\Rightarrow p = 4$

Now, the other equation becomes $4x^2 - qx + 9 = 0$

Here, $a = 4$, $b = -q$ and $c = 9$

Since the roots are equal, we have

$b^2 - 4ac = 0$

$\Rightarrow (-q)^2 - 4 \times 4 \times 9 = 0$

$\Rightarrow q^2 - 144 = 0$

$\Rightarrow q^2 = 144$

$\Rightarrow q = 12$

Hence, $p = 4$ and $q = 12$.

Question 33.

Solve : $\frac{x}{a} - \frac{a+b}{x} = \frac{b(a+b)}{ax}$

Solution:

$\frac{x}{a} - \frac{a+b}{x} = \frac{b(a+b)}{ax}$

$\Rightarrow \frac{x^2 - a^2 - ab}{ax} = \frac{ab + b^2}{ax}$

$\Rightarrow x^2 - a^2 - ab = ab + b^2$

$\Rightarrow x^2 = a^2 + b^2 + 2ab$

$\Rightarrow x^2 = (a+b)^2$

$\Rightarrow x = a + b$

Question 34.

Solve : $\left(\frac{1200}{x}+2\right)(x-10)-1200=60$

Solution:

$$\left(\frac{1200}{x}+2\right)(x-10)-1200=60$$
$$\Rightarrow 2\left(\frac{600}{x}+1\right)(x-10)=1260$$
$$\Rightarrow \left(\frac{600}{x}+1\right)(x-10)=630$$
$$\Rightarrow \left(\frac{600+x}{x}\right)(x-10)=630$$
$$\Rightarrow 600x-6000+x^2-10x=630x$$
$$\Rightarrow x^2-40x-6000=0$$
$$\Rightarrow x^2-100x+60x-6000=0$$
$$\Rightarrow x(x-100)+60(x-100)=0$$
$$\Rightarrow (x-100)(x+60)=0$$
$$\Rightarrow x-100=0 \text{ or } x+60=0$$
$$\Rightarrow x=100 \text{ or } x=-60$$

Question 35.

If -1 and 3 are the roots of $x^2 + px + q = 0$, find the values of p and q.

Solution:

Since -1 is a root of $x^2+px+q=0$, we have

$$(-1)^2+p(-1)+q=0$$
$$\Rightarrow 1-p+q=0$$
$$\Rightarrow -p+q=-1 \quad \ldots.(i)$$

Also, 3 is a root of $x^2+px+q=0$.

$$\Rightarrow (3)^2+p(3)+q=0$$
$$\Rightarrow 9+3p+q=0$$
$$\Rightarrow 3p+q=-9 \quad \ldots.(ii)$$

Subtracting equation (ii) from (i), we get

$-4p = 8$

$\Rightarrow p = -2$

$\Rightarrow -(-2) + q = -1$ ....[From (i)]

$\Rightarrow 2 + q = -1$

$\Rightarrow q = -3$

Hence, $p = -2$ and $q = -3$.

Exercise 5D

Question 1.

Solve each of the following equations using the formula :

(i) $x^2 - 6x = 27$ (ii) $x^2 - 10x + 21 = 0$

(iii) $x^2 + 6x - 10 = 0$ (iv) $x^2 + 2x - 6 = 0$

(v) $3x^2 + 2x - 1 = 0$ (vi) $2x^2 + 7x + 5 = 0$

(vii) $\frac{2}{3}x = -\frac{1}{6}x^2 - \frac{1}{3}$ (viii) $\frac{1}{15}x^2 + \frac{5}{3} = \frac{2}{3}x$

(ix) $x^2 - 6 = 2\sqrt{2}x$ (x) $\frac{4}{x} - 3 = \frac{5}{2x+3}$

(xi) $\frac{2x+3}{x+3} = \frac{x+4}{x+2}$ (xii) $\sqrt{6}x^2 - 4x - 2\sqrt{6} = 0$

(xiii) $\frac{2x}{x-4} + \frac{2x-5}{x-3} = 8\frac{1}{3}$ (xiv) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$

Solution:

(i) $x^2 - 6x = 27$

$\Rightarrow x^2 - 6x - 27 = 0$

Here $a = 1$, $b = -6$ and $c = -27$

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-27)}}{2(1)}$

$= \frac{6 \pm 12}{2} = \frac{6+12}{2}$ and $\frac{6-12}{2} = 9$ and -3

(ii)$x^2 - 10x + 21 = 0$

Here a=1, b=−10 and c=21

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(21)}}{2(1)}$

$= \frac{10 \pm 4}{2} = \frac{10+4}{2}$ and $\frac{10-4}{2} = 7$ and 3

(iii)$x^2 + 6x - 10 = 0$

Here a=1, b=6 and c=−10

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-10)}}{2(1)}$

$= \frac{-6 \pm \sqrt{76}}{2} = \frac{-6 + 2\sqrt{19}}{2}$ and $\frac{-6 - 2\sqrt{19}}{2} = -3 + \sqrt{19}$ and $-3 - \sqrt{19}$

(iv)$x^2 + 2x - 6 = 0$

Here a=1, b=2 and c=−6

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-6)}}{2(1)}$

$= \frac{-2 \pm \sqrt{28}}{2} = \frac{-2 \pm 2\sqrt{7}}{2} = -1 \pm \sqrt{7}$

(v)$3x^2 + 2x - 1 = 0$

Here a=3, b=2 and c=−1

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(-1)}}{2(3)}$

$$= \frac{-2 \pm 4}{6} = \frac{-2+4}{6} \text{ and } \frac{-2-4}{6} = \frac{1}{3} \text{ and } -1$$

(vi) $2x^2 + 7x + 5 = 0$

Here a=2, b=7 and c=5

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(7) \pm \sqrt{(7)^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{-7 \pm 3}{4} = \frac{-7+3}{4} \text{ and } \frac{-7-3}{4} = -1 \text{ and } -\frac{5}{2}$$

(vii) $\frac{2}{3}x = -\frac{1}{6}x^2 - \frac{1}{3}$

$\Rightarrow 4x = -x^2 - 2$

$\Rightarrow x^2 + 4x + 2 = 0$

Here a=1, b=4 and c=2

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{8}}{2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

(viii) $\frac{1}{15}x^2 + \frac{5}{3} = \frac{2}{3}x$

$\Rightarrow x^2 + 25 = 10x$

$\Rightarrow x^2 - 10x + 25 = 0$

Here a=1, b=– 10 and c=25

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{0}}{2} = 5$$

(ix) $x^2 - 6 = 2\sqrt{2}x$

$\Rightarrow \quad x^2 - 2\sqrt{2}x - 6 = 0$

Here $a=1$, $b=-2\sqrt{2}$ and $c=-6$

Then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(1)(-6)}}{2(1)}$

$= \dfrac{2\sqrt{2} \pm \sqrt{32}}{2} = \dfrac{2\sqrt{2} \pm 4\sqrt{2}}{2} = \dfrac{2\sqrt{2} + 4\sqrt{2}}{2}$ and $\dfrac{2\sqrt{2} - 4\sqrt{2}}{2}$

$= \dfrac{6\sqrt{2}}{2}$ and $\dfrac{-2\sqrt{2}}{2} = 3\sqrt{2}$ and $-\sqrt{2}$

(x) $\dfrac{4}{x} - 3 = \dfrac{5}{2x+3}$

$\Rightarrow \quad \dfrac{4-3x}{x} = \dfrac{5}{2x+3}$

$\Rightarrow (4-3x)(2x+3) = 5x$

$\Rightarrow 8x + 12 - 6x^2 - 9x = 5x$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$

Here $a=1$, $b=1$ and $c=-2$

Then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(1) \pm \sqrt{(1)^2 - 4(1)(-2)}}{2(1)}$

$= \dfrac{-1 \pm \sqrt{9}}{2} = \dfrac{-1 \pm 3}{2} = \dfrac{-1+3}{2}$ and $\dfrac{-1-3}{2} = 1$ and -2

(xi) $\dfrac{2x+3}{x+3} = \dfrac{x+4}{x+2}$

$\Rightarrow (2x+3)(x+2) = (x+3)(x+4)$

$\Rightarrow 2x^2 + 4x + 3x + 6 = x^2 + 4x + 3x + 12$

$\Rightarrow x^2 - 6 = 0$

Here $a=1$, $b=0$ and $c=-6$

Then $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{-(0)\pm\sqrt{(0)^2-4(1)(-6)}}{2(1)}$

$=\dfrac{0\pm\sqrt{24}}{2}=\dfrac{0\pm2\sqrt{6}}{2}=-\sqrt{6}$ and $\sqrt{6}$

(xii) $\sqrt{6}x^2-4x-2\sqrt{6}=0$

Here $a=\sqrt{6}$, $b=-4$ and $c=-2\sqrt{6}$

Then $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{-(-4)\pm\sqrt{(-4)^2-4(\sqrt{6})(-2\sqrt{6})}}{2(\sqrt{6})}$

$=\dfrac{4\pm\sqrt{64}}{2\sqrt{6}}=\dfrac{4\pm8}{2\sqrt{6}}=\dfrac{4+8}{2\sqrt{6}}$ and $\dfrac{4-8}{2\sqrt{6}}$

$=\dfrac{6}{\sqrt{6}}$ and $\dfrac{-2}{\sqrt{6}}=\sqrt{6}$ and $\dfrac{-\sqrt{6}}{3}$

(xiii) $\dfrac{2x}{x-4}+\dfrac{2x-5}{x-3}=8\dfrac{1}{3}$

$\Rightarrow\dfrac{2x(x-3)+(x-4)(2x-5)}{(x-4)(x-3)}=\dfrac{25}{3}$

$\Rightarrow\dfrac{2x^2-6x+2x^2-5x-8x+20}{x^2-3x-4x+12}=\dfrac{25}{3}$

$\Rightarrow\dfrac{4x^2-19x+20}{x^2-7x+12}=\dfrac{25}{3}$

$\Rightarrow 25x^2-175x+300=12x^2-57x+60$

$\Rightarrow 13x^2-118x+240=0$

Here $a=13$, $b=-118$ and $c=240$

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-118) \pm \sqrt{(-118)^2 - 4(13)(240)}}{2(13)}$

$= \frac{118 \pm \sqrt{1444}}{26} = \frac{118 \pm 38}{26}$

$= \frac{118 + 38}{26}$ and $\frac{118 - 38}{26} = 6$ and $\frac{40}{13}$

(xiv) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$

$\Rightarrow \frac{(x-1)(x-4) + (x-2)(x-3)}{(x-2)(x-4)} = \frac{10}{3}$

$\Rightarrow \frac{x^2 - 4x - x + 4 + x^2 - 3x - 2x + 6}{x^2 - 4x - 2x + 8} = \frac{10}{3}$

$\Rightarrow \frac{2x^2 - 10x + 10}{x^2 - 6x + 8} = \frac{10}{3}$

$\Rightarrow 10x^2 - 60x + 80 = 6x^2 - 30x + 30$

$\Rightarrow 4x^2 - 30x + 50 = 0$

$\Rightarrow 2x^2 - 15x + 25 = 0$

Here a=2, b=– 15 and c=25

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-15) \pm \sqrt{(-15)^2 - 4(2)(25)}}{2(2)}$

$= \frac{15 \pm \sqrt{25}}{4} = \frac{15 \pm 5}{4}$

$= \frac{15+5}{4}$ and $\frac{15-5}{4} = 5$ and $\frac{5}{2}$

Question 2.

Solve each of the following equations for x and give, in each case, your answer correct to one decimal place :

(i) $x^2 - 8x + 5 = 0$

(ii) $5x^2 + 10x - 3 = 0$

Solution:

(i) $x^2 - 8x + 5 = 0$

Here a=1, b=− 8 and c=5

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{44}}{2} = \frac{8 \pm 2\sqrt{11}}{2} = 4 \pm \sqrt{11} = 4 \pm 3.3 = 7.3 \text{ and } 0.7$$

(ii) $5x^2 + 10x - 3 = 0$

Here a=5, b=10 and c=− 3

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(10) \pm \sqrt{(10)^2 - 4(5)(-3)}}{2(5)}$$

$$= \frac{-10 \pm \sqrt{160}}{10} = \frac{-10 \pm 12.6}{10}$$

$$= \frac{-10 + 12.6}{10} \text{ and } \frac{-10 - 12.6}{10} = 0.26 \text{ and } -2.26 = 0.3 \text{ and } -2.3$$

Question 3(i).

Solve each of the following equations for x and give, in each case, your answer correct to two decimal places :

(i) $2x^2 - 10x + 5 = 0$

Solution:

$2x^2 - 10x + 5 = 0$

Here a=2, b=− 10 and c=5

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{10 \pm \sqrt{60}}{4} = \frac{10 \pm 7.75}{4}$$

$$= \frac{10 + 7.75}{4} \text{ and } \frac{10 - 7.75}{4} = 4.44 \text{ and } 0.56$$

Question 3(ii).
Solve each of the following equations for x and give, in each case, your answer correct to two decimal places :
$4x + 6/x + 13 = 0$

Solution:

$$4x+\frac{6}{x}+13=0$$

$$\Rightarrow 4x^2+6+13x=0$$

$$\Rightarrow 4x^2+13x+6=0$$

Here $a=4$, $b=13$ and $c=6$

$$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(13)\pm\sqrt{(13)^2-4(4)(6)}}{2(4)}$$

$$=\frac{-13\pm\sqrt{73}}{8}=\frac{-13\pm 8.54}{8}$$

$$=\frac{-13+8.54}{8}\text{ and }\frac{-13-8.54}{8}=-0.56\text{ and }-2.69$$

Question 3(iii).
Solve each of the following equations for x and give, in each case, your answer correct to two decimal places :
$x^2 - 3x - 9 = 0$

Solution:

$$x^2-3x-9=0$$

Here $a=1$, $b=-3$ and $c=-9$

$$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-9)}}{2(1)}$$

$$=\frac{3\pm\sqrt{45}}{2}=\frac{3\pm 6.70}{2}$$

$$=\frac{3+6.70}{2}\text{ and }\frac{3-6.70}{2}=4.85\text{ and }-1.85$$

Question 3(iv).
Solve each of the following equations for x and give, in each case, your answer correct to two decimal places :
$x^2 - 5x - 10 = 0$

Solution:

$$x^2 - 5x - 10 = 0$$

Here, $a = 1$, $b = -5$ and $c = -10$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times (-10)}}{2 \times 1}$$

$$= \frac{5 \pm \sqrt{25 + 40}}{2}$$

$$= \frac{5 \pm \sqrt{65}}{2}$$

$$= \frac{5 \pm 8.06}{2}$$

$$\therefore x = \frac{5 + 8.06}{2} \text{ or } x = \frac{5 - 8.06}{2}$$

$$\Rightarrow x = \frac{13.06}{2} \text{ or } x = -\frac{3.06}{2}$$

$$\Rightarrow x = 6.53 \text{ or } x = -1.53$$

Question 4.
Solve each of the following equations for x and give, in each case, your answer correct to 3 decimal places :
(i) $3x^2 - 12x - 1 = 0$
(ii) $x^2 - 16x + 6 = 0$
(iii) $2x^2 + 11x + 4 = 0$

Solution:

(i) $3x^2 - 12x - 1 = 0$

Here $a=3$, $b=-12$ and $c=-1$

$$\therefore x=\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$=\frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(-1)}}{2(3)}$$

$$=\frac{12 \pm \sqrt{156}}{6} = \frac{12 \pm 12.489}{6}$$

$$=\frac{12 + 12.489}{6} \text{ and } \frac{12 - 12.489}{6} = 4.082 \text{ and } -0.082$$

(ii) $x^2 - 16x + 6 = 0$

Here $a=1$, $b=-16$ and $c=6$

$$\therefore x=\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$=\frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(6)}}{2(1)}$$

$$=\frac{16 \pm \sqrt{232}}{2} = \frac{16 \pm 15.231}{2}$$

$$=\frac{16 + 15.231}{2} \text{ and } \frac{16 - 15.231}{2} = 15.616 \text{ and } 0.384$$

(iii) $2x^2 + 11x + 4 = 0$

Here $a=2$, $b=11$ and $c=4$

$$\therefore x=\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$=\frac{-(11) \pm \sqrt{(11)^2 - 4(2)(4)}}{2(2)}$$

$$=\frac{-11 \pm \sqrt{89}}{4} = \frac{-11 \pm 9.433}{4}$$

$$=\frac{-11 + 9.433}{4} \text{ and } \frac{-11 - 9.433}{4} = -0.392 \text{ and } -5.108$$

Question 5.
Solve:
(i) $x^4 - 2x^2 - 3 = 0$
(ii) $x^4 - 10x^2 + 9 = 0$

Solution:

(i) $x^4 - 2x^2 - 3 = 0$

$\Rightarrow x^4 - 3x^2 + x^2 - 3 = 0$

$\Rightarrow x^2(x^2 - 3) + 1(x^2 - 3) = 0$

$\Rightarrow (x^2 - 3)(x^2 + 1) = 0$

If $x^2 - 3 = 0$ or $x^2 + 1 = 0$

$\Rightarrow x^2 = 3$ or $x^2 = -1$(reject)

$\Rightarrow x = \pm\sqrt{3}$

(ii) $x^4 - 10x^2 + 9 = 0$

$\Rightarrow x^4 - 9x^2 - x^2 + 9 = 0$

$\Rightarrow x^2(x^2 - 9) - 1(x^2 - 9) = 0$

$\Rightarrow (x^2 - 9)(x^2 - 1) = 0$

If $x^2 - 9 = 0$ or $x^2 - 1 = 0$

$\Rightarrow x^2 = 9$ or $x^2 = 1$

$\Rightarrow x = \pm 3$ or $x = \pm 1$

Question 6.
Solve :
(i) $(x^2 - x)^2 + 5(x^2 - x) + 4 = 0$
(ii) $(x^2 - 3x)^2 - 16(x^2 - 3x) - 36 = 0$

Solution:

(i) $(x^2 - x)^2 + 5(x^2 - x) + 4 = 0$

Let $x^2 - x = y$

Then $y^2 + 5y + 4 = 0$

$\Rightarrow y^2 + 4y + y + 4 = 0$

$\Rightarrow y(y + 4) + 1(y + 4) = 0$

$\Rightarrow (y + 4)(y + 1) = 0$

If $y+4=0$ or $y+1=0$

$\Rightarrow x^2 - x + 4 = 0$ or $x^2 - x + 1 = 0$

$\Rightarrow x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(4)}}{2(1)}$ or $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$

$\Rightarrow x = \frac{1 \pm \sqrt{-15}}{2}$ (reject) or $x = \frac{1 \pm \sqrt{-3}}{2}$ (reject)

∴ Given equation has no real solution.

(ii) $(x^2 - 3x)^2 - 16(x^2 - 3x) - 36 = 0$

Let $x^2 - 3x = y$

Then $y^2 - 16y - 36 = 0$

$\Rightarrow y^2 - 18y + 2y - 36 = 0$

$\Rightarrow y(y - 18) + 2(y - 18) = 0$

$\Rightarrow (y - 18)(y + 2) = 0$

If $y - 18=0$ or $y+2=0$

$\Rightarrow x^2 - 3x - 18 = 0$ or $x^2 - 3x + 2 = 0$

$\Rightarrow x^2 - 6x + 3x - 18 = 0$ or $x^2 - 2x - x + 2 = 0$

$\Rightarrow x(x - 6) + 3(x - 6) = 0$ or $x(x - 2) - 1(x - 2) = 0$

$\Rightarrow (x - 6)(x + 3) = 0$ or $(x - 2)(x - 1) = 0$

If $x - 6 = 0$ or $x+3=0$ or $x - 2=0$ or $x - 1=0$

then $x=6$ or $x=-3$ or $x=2$ or $x=1$

Question 7.

Solve :

(i) $\sqrt{\frac{x}{x-3}}+\sqrt{\frac{x-3}{x}}=\frac{5}{2}$

(ii) $\left(\frac{2x-3}{x-1}\right)-4\left(\frac{x-1}{2x-3}\right)=3$

(iii) $\left(\frac{3x+1}{x+1}\right)+\left(\frac{x+1}{3x+1}\right)=\frac{5}{2}$

Solution:

(i) $\sqrt{\frac{x}{x-3}}+\sqrt{\frac{x-3}{x}}=\frac{5}{2}$

Let $\sqrt{\frac{x}{x-3}}=y$

Then $y+\frac{1}{y}=\frac{5}{2}$

$\Rightarrow \frac{y^2+1}{y}=\frac{5}{2}$

$\Rightarrow 2y^2+2=5y$

$\Rightarrow 2y^2-5y+2=0$

$\Rightarrow 2y^2-4y-y+2=0$

$\Rightarrow 2y(y-2)-1(y-2)=0$

$\Rightarrow (y-2)(2y-1)=0$

If $y-2=0$ or $2y-1=0$

then $y=2$ or $y=\frac{1}{2}$

$\Rightarrow \sqrt{\frac{x}{x-3}}=2$ or $\sqrt{\frac{x}{x-3}}=\frac{1}{2}$

$\Rightarrow \frac{x}{x-3}=4$ or $\frac{x}{x-3}=\frac{1}{4}$

$\Rightarrow x=4$ or $x=-1$

(ii) $\left(\frac{2x-3}{x-1}\right)-4\left(\frac{x-1}{2x-3}\right)=3$

Let $\frac{2x-3}{x-1}=y$

then $y-\frac{4}{y}=3$

$\Rightarrow \frac{y^2-4}{y}=3$

$\Rightarrow y^2-4=3y$

$\Rightarrow y^2-3y-4=0$

$\Rightarrow y^2-4y+y-4=0$

$\Rightarrow y(y-4)+1(y-4)=0$

$\Rightarrow (y-4)(y+1)=0$

If $y-4=0$ or $y+1=0$

then $y=4$ or $y=-1$

$\Rightarrow \frac{2x-3}{x-1}=4$ or $\frac{2x-3}{x-1}=-1$

$\Rightarrow 4x-4=2x-3$ or $2x-3=-x+1$

$\Rightarrow 2x=1$ or $3x=4$

$\Rightarrow x=\frac{1}{2}$ or $x=\frac{4}{3}=1\frac{1}{3}$

(iii) $\left(\frac{3x+1}{x+1}\right)+\left(\frac{x+1}{3x+1}\right)=\frac{5}{2}$

Let $\frac{3x+1}{x+1}=y$

then $y+\frac{1}{y}=\frac{5}{2}$

$\Rightarrow \frac{y^2+1}{y}=\frac{5}{2}$

$\Rightarrow 2y^2+2=5y$

$\Rightarrow 2y^2-5y+2=0$

$\Rightarrow 2y^2 - 4y - y + 2 = 0$

$\Rightarrow 2y(y-2) - 1(y-2) = 0$

$\Rightarrow (y-2)(2y-1) = 0$

If $y-2=0$ or $2y-1=0$

then $y=2$ or $y=\frac{1}{2}$

$\Rightarrow \frac{3x+1}{x+1} = 2$ or $\frac{3x+1}{x+1} = \frac{1}{2}$

$\Rightarrow 3x+1 = 2x+2$ or $6x+2 = x+1$

$\Rightarrow x = 1$ or $5x = -1$

$\Rightarrow x = 1$ or $x = \frac{-1}{5}$

Question 8.

Solve the equation $2x - \frac{1}{x} = 7$. Write your answer correct to two decimal places.

Solution:

$2x - \frac{1}{x} = 7$

$\Rightarrow \frac{2x^2 - 1}{x} = 7$

$\Rightarrow 2x^2 - 1 = 7x$

$\Rightarrow 2x^2 - 7x - 1 = 0$

Here $a=2$, $b=-7$ and $c=-1$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-1)}}{2(2)}$

$= \frac{7 \pm \sqrt{57}}{4} = \frac{7 \pm 7.55}{4}$

$= \frac{7+7.55}{4}$ and $\frac{7-7.55}{4} = 3.64$ and -0.14

Question 9.

Solve the following equation and give your answer correct to 3 significant figures:

$5x^2 - 3x - 4 = 0$

Solution:

Consider the given equation:

$5x^2 - 3x - 4 = 0$

Using quadratic formula, we have,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 5 \times (-4)}}{2 \times 5}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 + 80}}{2 \times 5}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{89}}{10}$$

$$\Rightarrow x = \frac{3 \pm 9.434}{10}$$

$\Rightarrow x = 1.243$ or $x = -0.643$

Question 10.
Solve for x using the quadratic formula. Write your answer correct to two significant figures.

$(x - 1)^2 - 3x + 4 = 0$

Solution:

$(x-1)^2 - 3x + 4 = 0$

$\Rightarrow x^2 - 2x + 1 - 3x + 4 = 0$

$\Rightarrow x^2 - 5x + 5 = 0$

Here, $a = 1$, $b = -5$ and $c = 5$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$$= \frac{5 \pm \sqrt{25 - 20}}{2}$$

$$= \frac{5 \pm \sqrt{5}}{2}$$

$$= \frac{5 \pm 2.24}{2}$$

$$\therefore x = \frac{5 + 2.24}{2} \text{ or } x = \frac{5 - 2.24}{2}$$

$$\Rightarrow x = \frac{7.24}{2} \text{ or } x = \frac{2.76}{2}$$

$$\Rightarrow x = 3.6 \text{ or } x = 1.4$$

Question 11.
Solve the quadratic equation $x^2 - 3(x+3) = 0$; Give your answer correct to two significant figures.

Solution:

$x^2 - 3(x + 3) = 0$

$\Rightarrow x^2 - 3x - 9 = 0$

Comparing with $ax^2 + bx + c$, we get

$a = 1, b = -3, c = -9$

Now, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)}$

$\Rightarrow x = \frac{3 \pm \sqrt{9 + 36}}{2}$

$\Rightarrow x = \frac{3 \pm \sqrt{45}}{2}$

$\Rightarrow x = \frac{3 \pm \sqrt{9 \times 5}}{2}$

$\Rightarrow x = \frac{3 \pm 3\sqrt{5}}{2}$

$$\Rightarrow x = \frac{3+3\sqrt{5}}{2} \text{ or } x = \frac{3-3\sqrt{5}}{2}$$
$$\Rightarrow x = \frac{3+3\times 2.236}{2} \text{ or } x = \frac{3-3\times 2.236}{2}$$
$$\Rightarrow x = \frac{3+6.708}{2} \text{ or } x = \frac{3-6.708}{2}$$
$$\Rightarrow x = \frac{9.708}{2} \text{ or } x = \frac{-3.708}{2}$$
$$\Rightarrow x = 4.854 \text{ or } x = -1.854$$
$$\Rightarrow x = 4.9 \text{ or } x = -1.9$$

Exercise 5E

Question 1.

Solve:
$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0;\ x \neq 3,\ x \neq -\frac{3}{2}$$

Solution:

$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0;\ x \neq 3,\ x \neq -\frac{3}{2}$$
$$\Rightarrow \frac{2x(2x+3)+1(x-3)+3x+9}{(x-3)(2x+3)} = 0$$
$$\Rightarrow 4x^2 + 6x + x - 3 + 3x + 9 = 0$$
$$\Rightarrow 4x^2 + 10x + 6 = 0$$
$$\Rightarrow 4x^2 + 4x + 6x + 6 = 0$$
$$\Rightarrow 4x(x+1) + 6(x+1) = 0$$
$$\Rightarrow (x+1)(4x+6) = 0$$
$$\Rightarrow \quad x+1=0 \quad \text{or} \quad 4x+6=0$$
$$\Rightarrow \quad x=-1 \quad \text{or} \quad x=\frac{-6}{4}=\frac{-3}{2}(\text{reject})$$

Question 2.

Solve: $(2x+3)^2=81$

Solution:

$$(2x+3)^2 = 81$$
$$\Rightarrow 2x+3 = \pm 9$$
$$\Rightarrow 2x+3 = 9 \quad \text{and} \quad 2x+3 = -9$$
$$\Rightarrow 2x = 6 \quad \text{and} \quad 2x = -12$$
$$\Rightarrow x = 3 \quad \text{and} \quad x = -6$$

Question 3.
Solve: $a^2x^2 - b^2 = 0$

Solution:

$$a^2x^2 - b^2 = 0$$
$$\Rightarrow (ax)^2 - b^2 = 0$$
$$\Rightarrow (ax+b)(ax-b) = 0$$
$$\text{If } ax+b=0 \quad \text{and} \quad ax-b=0$$
$$\text{then } x = \frac{-b}{a} \quad \text{and} \quad x = \frac{b}{a}$$

Question 4.
Solve: $x^2 - \frac{11}{4}x + \frac{15}{8} = 0$

Solution:

$$x^2 - \frac{11}{4}x + \frac{15}{8} = 0$$
$$\Rightarrow \frac{8x^2 - 22x + 15}{8} = 0$$
$$\Rightarrow 8x^2 - 22x + 15 = 0$$
$$\Rightarrow 8x^2 - 12x - 10x + 15 = 0$$
$$\Rightarrow 4x(2x-3) - 5(2x-3) = 0$$
$$\Rightarrow (2x-3)(4x-5) = 0$$
$$\Rightarrow \quad 2x-3 = 0 \quad \text{or} \quad 4x-5 = 0$$
$$\Rightarrow \quad x = \frac{3}{2} \quad \text{or} \quad x = \frac{5}{4}$$

Question 5.

$x + \frac{4}{x} = -4; x \neq 0$

Solution:

$$x + \frac{4}{x} = -4$$
$$\Rightarrow \frac{x^2 + 4}{x} = -4$$
$$\Rightarrow x^2 + 4 = -4x$$
$$\Rightarrow x^2 + 4x + 4 = 0$$
$$\Rightarrow (x + 2)^2 = 0$$
$$\Rightarrow x + 2 = 0$$
$$\Rightarrow x = -2$$

Question 6.
Solve: $2x^4 - 5x^2 + 3 = 0$

Solution:

$$2x^4 - 5x^2 + 3 = 0$$
$$\Rightarrow 2x^4 - 3x^2 - 2x^2 + 3 = 0$$
$$\Rightarrow x^2(2x^2 - 3) - 1(2x^2 - 3) = 0$$
$$\Rightarrow (2x^2 - 3)(x^2 - 1) = 0$$
If $2x^2 - 3 = 0$ or $x^2 - 1 = 0$
then $x^2 = \frac{3}{2}$ or $x^2 = 1$
$\Rightarrow$ $x = \pm\sqrt{\frac{3}{2}}$ or $x = \pm 1$

Question 7.
Solve: $x^4 - 2x^2 - 3 = 0$.

Solution:

$x^4 - 2x^2 - 3 = 0$

$\Rightarrow x^4 - 3x^2 + x^2 - 3 = 0$

$\Rightarrow x^2(x^2 - 3) + 1(x^2 - 3) = 0$

$\Rightarrow (x^2 - 3)(x^2 + 1) = 0$

If $x^2 - 3 = 0$ or $x^2 + 1 = 0$

then $x^2 = 3$ or $x^2 = -1$(reject)

$\Rightarrow x = \pm\sqrt{3}$

Question 8.

Solve : $9\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) - 52 = 0$

Solution:

$9\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) - 52 = 0$

Let $x + \frac{1}{x} = y$

squaring on both sides

$x^2 + \frac{1}{x^2} + 2 = y^2$

$\Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$

Puting these values in the given equation

$9(y^2 - 2) - 9y - 52 = 0$

$\Rightarrow 9y^2 - 18 - 9y - 52 = 0$

$\Rightarrow 9y^2 - 9y - 70 = 0$

$\Rightarrow 9y^2 - 30y + 21y - 70 = 0$

$\Rightarrow 3y(3y - 10) + 7(3y - 10) = 0$

$\Rightarrow (3y-10)(3y+7)=0$

$\Rightarrow \quad 3y-10=0 \quad \text{or} \quad 3y+7=0$

$\Rightarrow \quad y=\frac{10}{3} \quad \text{or} \quad y=\frac{-7}{3}$

$\Rightarrow x+\frac{1}{x}=\frac{10}{3} \quad \text{or} \quad x+\frac{1}{x}=\frac{-7}{3}$

$\Rightarrow \frac{x^2+1}{x}=\frac{10}{3} \quad \text{or} \quad \frac{x^2+1}{x}=\frac{-7}{3}$

$\Rightarrow 3x^2-10x+3=0 \quad \text{or} \quad 3x^2+7x+3=0$

$\Rightarrow 3x^2-9x-x+3=0 \quad \text{or} \quad x=\frac{-7\pm\sqrt{(-7)^2-4(3)(3)}}{2(3)}$

$\Rightarrow 3x(x-3)-1(x-3)=0 \quad \text{or} \quad x=\frac{-7\pm\sqrt{13}}{6}$

$\Rightarrow (x-3)(3x-1)=0$

$\Rightarrow \quad x=3 \text{ and } x=\frac{1}{3}$

$\Rightarrow (3y-10)(3y+7)=0$

$\Rightarrow \quad 3y-10=0 \quad \text{or} \quad 3y+7=0$

$\Rightarrow \quad y=\frac{10}{3} \quad \text{or} \quad y=\frac{-7}{3}$

$\Rightarrow x+\frac{1}{x}=\frac{10}{3} \quad \text{or} \quad x+\frac{1}{x}=\frac{-7}{3}$

$\Rightarrow \frac{x^2+1}{x}=\frac{10}{3} \quad \text{or} \quad \frac{x^2+1}{x}=\frac{-7}{3}$

$\Rightarrow 3x^2-10x+3=0 \quad \text{or} \quad 3x^2+7x+3=0$

$\Rightarrow 3x^2-9x-x+3=0 \quad \text{or} \quad x=\frac{-7\pm\sqrt{(-7)^2-4(3)(3)}}{2(3)}$

$\Rightarrow 3x(x-3)-1(x-3)=0 \quad \text{or} \quad x=\frac{-7\pm\sqrt{13}}{6}$

$\Rightarrow (x-3)(3x-1)=0$

$\Rightarrow \quad x=3 \text{ and } x=\frac{1}{3}$

Question 9.

Solve : $2\left(x^2+\frac{1}{x^2}\right)-\left(x+\frac{1}{x}\right)=11$

Solution:

$$2\left(x^2+\frac{1}{x^2}\right)-\left(x+\frac{1}{x}\right)=11$$

Let $x+\frac{1}{x}=y$

squaring on both sides

$$x^2+\frac{1}{x^2}+2=y^2$$

$$\Rightarrow x^2+\frac{1}{x^2}=y^2-2$$

Puting these values in the given equation

$$2(y^2-2)-y=11$$

$$\Rightarrow 2y^2-4-y-11=0$$

$$\Rightarrow 2y^2-y-15=0$$

$$\Rightarrow 2y^2-6y+5y-15=0$$

$$\Rightarrow 2y(y-3)+5(y-3)=0$$

$$\Rightarrow (y-3)(2y+5)=0$$

If $y-3=0$ or $2y+5=0$

then $y=3$ or $y=\frac{-5}{2}$

$$\Rightarrow x+\frac{1}{x}=3 \quad \text{or} \quad x+\frac{1}{x}=\frac{-5}{2}$$

$$\Rightarrow \frac{x^2+1}{x}=3 \quad \text{or} \quad \frac{x^2+1}{x}=\frac{-5}{2}$$

$$\Rightarrow x^2-3x+1=0 \quad \text{or} \quad 2x^2+5x+2=0$$

$$\Rightarrow x=\frac{-3\pm\sqrt{(-3)^2-4(1)(1)}}{2(1)} \quad \text{or} \quad 2x^2+4x+x+2=0$$

$\Rightarrow x=\frac{-3\pm\sqrt{5}}{2}$ or $2x(x+2)+1(x+2)=0$

or $(x+2)(2x+1)=0$

then $x=-2$ and $x=\frac{-1}{2}$

Question 10.

Solve : $\left(x^2+\frac{1}{x^2}\right)-3\left(x-\frac{1}{x}\right)-2=0$

Solution:

$\left(x^2+\frac{1}{x^2}\right)-3\left(x-\frac{1}{x}\right)-2=0$

Let $x-\frac{1}{x}=y$

squaring on both sides

$x^2+\frac{1}{x^2}-2=y^2$

$\Rightarrow x^2+\frac{1}{x^2}=y^2+2$

Puting these values in the given equation

$(y^2+2)-3y-2=0$

$\Rightarrow y^2-3y=0$

$\Rightarrow y(y-3)=0$

If $y=0$ or $y-3=0$

then $y=0$ or $y=3$

$\Rightarrow x-\frac{1}{x}=0$ or $x-\frac{1}{x}=3$

$\Rightarrow \frac{x^2-1}{x}=0$ or $\frac{x^2-1}{x}=3$

$\Rightarrow x^2-1=0$ or $x^2-3x-1=0$

$\Rightarrow (x+1)(x-1)=0$ or $x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-1)}}{2(1)}$

$\Rightarrow x=-1$ and $x=1$ or $x=\frac{3\pm\sqrt{13}}{2}$

Question 11.
Solve : $(x^2 + 5x + 4)(x^2 + 5x + 6) = 120$

Solution:

$(x^2 + 5x + 4)(x^2 + 5x + 6) = 120$

Let $x^2 + 5x = y$

then $(y+4)(y+6)=120$

$\Rightarrow y^2 + 6y + 4y + 24 - 120 = 0$

$\Rightarrow y^2 + 10y - 96 = 0$

$\Rightarrow y^2 + 16y - 6y - 96 = 0$

$\Rightarrow y(y + 16) - 6(y + 16) = 0$

$\Rightarrow (y + 16)(y - 6) = 0$

then $y = -16$ or $y=6$

$\Rightarrow x^2 + 5x + 16 = 0$ or $x^2 + 5x - 6 = 0$

$\Rightarrow x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(16)}}{2(1)}$ or $x^2 + 6x - x - 6 = 0$

$\Rightarrow x = \frac{-5 \pm \sqrt{-39}}{2}$ or $x(x+6)-1(x+6)=0$

(reject) or $(x+6)(x-1)=0$

then $x=-6$ and $x=1$

Question 12.
Solve each of the following equations, giving answer upto two decimal places.
(i) $x^2 - 5x - 10=0$ (ii) $3x^2 - x - 7 =0$

Solution:

(i)$x^2 - 5x - 10 = 0$

Here $a = 1$, $b = -5$ and $c = -10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-10)}}{2(1)}$$

$$\Rightarrow x = \frac{5 \pm \sqrt{65}}{2} = \frac{5 \pm 8.06}{2}$$

$$\Rightarrow x = \frac{13.06}{2} \text{ and } \frac{-3.06}{2} = 6.53 \text{ and } -1.53$$

(ii)$3x^2 - x - 7 = 0$

Here $a = 3$, $b = -1$ and $c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-7)}}{2(3)}$$

$$\Rightarrow x = \frac{1 \pm \sqrt{85}}{6} = \frac{1 \pm 9.22}{6}$$

$$\Rightarrow x = \frac{10.22}{6} \text{ and } \frac{-8.22}{6} = 1.70 \text{ and } -1.37$$

Question 13.

Solve : $\left(\frac{x}{x+2}\right)^2 - 7\left(\frac{x}{x+2}\right) + 12 = 0;\ x \neq -2$

Solution:

$$\left(\frac{x}{x+2}\right)^2 - 7\left(\frac{x}{x+2}\right) + 12 = 0;\ x \neq -2$$

Let $\frac{x}{x+2} = y$

then $y^2 - 7y + 12 = 0$

$\Rightarrow y^2 - 4y - 3y + 12 = 0$

$\Rightarrow y(y-4) - 3(y-4) = 0$

$\Rightarrow (y-4)(y-3)=0$

then $y=4$ and $y=3$

$\Rightarrow \frac{x}{x+2}=4$ and $\frac{x}{x+2}=3$

$\Rightarrow 4x+8=x$ and $3x+6=x$

$\Rightarrow x=\frac{-8}{3}$ and $x=-3$

Question 14.

Solve :

(i) $x^2 - 11x - 12 = 0$; when $x \in N$

(ii) $x^2 - 4x - 12 = 0$; when $x \in I$

(iii) $2x^2 - 9x + 10 = 0$; when $x \in Q$

Solution:

(i) $x^2 - 11x - 12 = 0$

$\Rightarrow x^2 - 12x + x - 12 = 0$

$\Rightarrow x(x-12) + 1(x-12) = 0$

$\Rightarrow (x-12)(x+1) = 0$

then $x = 12$ and $x = -1$

Since $x \in N$, then $x = 12$

(ii) $x^2 - 4x - 12 = 0$

$\Rightarrow x^2 - 6x + 2x - 12 = 0$

$\Rightarrow x(x-6) + 2(x-6) = 0$

$\Rightarrow (x-6)(x+2) = 0$

then $x = 6$ and $x = -2$

Since $x \in I$, then $x = 6$ and -2

(iii) $2x^2 - 9x + 10 = 0$

$\Rightarrow 2x^2 - 5x - 4x - 10 = 0$

$\Rightarrow x(2x-5) - 2(2x-5) = 0$

$\Rightarrow (2x-5)(x-2) = 0$

then $x = \frac{5}{2}$ and $x = 2$

Since $x \in Q$, then $x = \frac{5}{2}$ and 2

Question 15.
Solve : $(a + b)^2x^2 - (a + b)x - 6 = 0;\ a + b \neq 0$

Solution:

$$(a+b)^2x^2-(a+b)x-6=0;\ a+b\neq 0$$
$$\Rightarrow (a+b)^2x^2-3(a+b)x+2(a+b)x-6=0$$
$$\Rightarrow (a+b)x[(a+b)x-3]+2[(a+b)x-3]=0$$
$$\Rightarrow [(a+b)x-3][(a+b)x+2]=0$$
$$\Rightarrow \quad (a+b)x-3=0 \quad \text{or} \quad (a+b)x+2=0$$
$$\Rightarrow \quad x=\frac{3}{a+b} \qquad \text{or} \qquad x=\frac{-2}{a+b}$$

Question 16.
Solve : $\frac{1}{p}+\frac{1}{q}+\frac{1}{x}=\frac{1}{x+p+q}$

Solution:

$$\frac{1}{p}+\frac{1}{q}+\frac{1}{x}=\frac{1}{x+p+q}$$
$$\Rightarrow \frac{1}{p}+\frac{1}{q}+\frac{1}{x}-\frac{1}{x+p+q}=0$$
$$\Rightarrow \frac{q+p}{pq}+\frac{x+p+q-x}{x(x+p+q)}=0$$
$$\Rightarrow \frac{q+p}{pq}+\frac{p+q}{x(x+p+q)}=0$$
$$\Rightarrow (p+q)\left[\frac{1}{pq}+\frac{1}{x^2+px+qx}\right]=0$$
$$\Rightarrow (p+q)\left[\frac{x^2+px+qx+pq}{pq(x^2+px+qx)}\right]=0$$
$$\Rightarrow x^2+px+qx+pq=0$$
$$\Rightarrow x(x+p)+q(x+p)=0$$
$$\Rightarrow (x+p)(x+q)=0$$
$$\Rightarrow \quad x=-p \text{ and } x=-q$$

Question 17.

Solve :

(i) $x(x+1)+(x+2)(x+3)=42$

(ii) $\frac{1}{x+1}-\frac{2}{x+2}=\frac{3}{x+3}-\frac{4}{x+4}$

Solution:

(i) $x(x+1)+(x+2)(x+3)=42$

$\Rightarrow x^2+x+x^2+3x+2x+6-42=0$

$\Rightarrow 2x^2+6x-36=0$

$\Rightarrow 2x^2+12x-6x-36=0$

$\Rightarrow 2x(x+6)-6(x+6)=0$

$\Rightarrow (x+6)(2x-6)=0$

If $x+6=0$ or $2x-6=0$

then $x=-6$ or $x=3$

(ii) $\frac{1}{x+1}-\frac{2}{x+2}=\frac{3}{x+3}-\frac{4}{x+4}$

$\Rightarrow \frac{1(x+2)-2(x+1)}{(x+1)(x+2)}=\frac{3(x+4)-4(x+3)}{(x+3)(x+4)}$

$\Rightarrow \frac{-x}{x^2+3x+2}=\frac{-x}{x^2+7x+12}$

$\Rightarrow -x\left[x^2+3x+2=x^2+7x+12\right]$

$\Rightarrow -x[-4x=10]$

$\Rightarrow x=0$ and $x=\frac{-10}{4}=-2.5$

Question 18.

For each equation, given below, find the value of m so that the equation has equal roots. Also, find the solution of each equation :

(i) $(m-3)x^2-4x+1=0$

(ii) $3x^2+12x+(m+7)=0$

(iii) $x^2-(m+2)x+(m+5)=0$

Solution:

(i)$(m-3)x^2 - 4x + 1 = 0$

Here a=(m-3), b= -4 and c= 1

Given equation has equal roots

then D=0

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (-4)^2 - 4(m-3)(1) = 0$

$\Rightarrow 16 - 4m + 12 = 0$

$\Rightarrow -4m = -28$

$\Rightarrow m = 7$

Put value of m in given equation

$4x^2 - 4x + 1 = 0$

$\Rightarrow (2x-1)^2 = 0$

$\Rightarrow 2x - 1 = 0$

$\Rightarrow x = \frac{1}{2}$

(ii)$3x^2 + 12x + (m+7) = 0$

Here a=3, b= 12 and c= m+7

Given equation has equal roots

then D=0

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (12)^2 - 4(3)(m+7) = 0$

$\Rightarrow 144 - 12m - 84 = 0$

$\Rightarrow -12m = -60$

$\Rightarrow m = 5$

Put value of m in given equation

$3x^2 + 12x + 12 = 0$

$\Rightarrow x^2 + 4x + 4 = 0$

$\Rightarrow (x + 2)^2 = 0$

$\Rightarrow x + 2 = 0$

$\Rightarrow x = -2$

(iii) $x^2 - (m + 2)x + (m + 5) = 0$

Here a=1, b= – (m+2) and c= m+5

Given equation has equal roots

then D=0

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow [-(m + 2)]^2 - 4(1)(m + 5) = 0$

$\Rightarrow m^2 + 4m + 4 - 4m - 20 = 0$

$\Rightarrow m^2 - 16 = 0$

$\Rightarrow m^2 = 16$

$\Rightarrow m = \pm 4$

Put value of m in given equation

$x^2 - 6x + 9 = 0$	or	$x^2 + 2x + 1 = 0$
$\Rightarrow (x - 3)^2 = 0$	or	$(x+1)^2 = 0$
$\Rightarrow x - 3 = 0$	or	$x+1=0$
$\Rightarrow x = 3$	or	$x = -1$

Question 19.

Without solving the following quadratic equation, find the value of p for which the roots are equal.

$px^2 - 4x + 3=0$

Solution:

$px^2 - 4x + 3 = 0$

Here $a=p$, $b=-4$ and $c=3$

Given equation has equal roots

then $D=0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow [-4]^2 - 4(p)(3) = 0$

$\Rightarrow 16 - 12p = 0$

$\Rightarrow -12p = -16$

$\Rightarrow p = \frac{-16}{-12} = \frac{4}{3}$

Question 20.

Without solving the following quadratic equation, find the value of 'm' for which the given equation has real and equal roots.

$x^2 + 2(m - 1)x + (m + 5) = 0$

Solution:

Consider the given equation:

$x^2 + 2(m-1)x + (m+5) = 0$

The nature of the roots of a quadratic equation

$ax^2 + bx + c = 0$, depends entirely on the

value of its discriminant $b^2 - 4ac$.

If a, b and c are real numbers and $a \neq 0$,

then discriminant:

(i) $b^2 - 4ac = 0 \Rightarrow$ the roots are real and equal.

(ii) $b^2 - 4ac > 0 \Rightarrow$ the roots are real and unequal.

(i) $b^2 - 4ac < 0 \Rightarrow$ the roots are imaginary (not real).

Since the roots of the given equation are real and equal,

we have,

$b^2 - 4ac = 0$

$\Rightarrow (2(m-1))^2 - 4 \times 1 \times (m+5) = 0$

$\Rightarrow 4\left(m^2+1-2m\right)-4(m+5)=0$

$\Rightarrow 4m^2+4-8m-4m-20=0$

$\Rightarrow 4m^2-12m-16=0$

$\Rightarrow m^2-3m-4=0$

$\Rightarrow m^2-4m+m-4=0$

$\Rightarrow m(m-4)+1(m-4)=0$

$\Rightarrow (m+1)(m-4)=0$

$\Rightarrow m+1=0$ or $m-4=0$

$\Rightarrow m=-1$ or $m=4$

Exercise 5F

Solution 1(i)

Given: $(x+5)(x-5)=24$

$\Rightarrow x^2-5^2=24$ .... since $(a-b)(a+b)=a^2-b^2$

$\Rightarrow x^2-25=24$

$\Rightarrow x^2=49$

$\Rightarrow x=\pm 7$

Solution 1(ii)

Given: $3x^2-2\sqrt{6}x+2=0$

$\Rightarrow 3x^2-\sqrt{6}x-\sqrt{6}x+2=0$

$\Rightarrow \sqrt{3}x(\sqrt{3}x-\sqrt{2})-\sqrt{2}(\sqrt{3}x-\sqrt{2})=0$

$\Rightarrow (\sqrt{3}x-\sqrt{2})(\sqrt{3}x-\sqrt{2})=0$

$\Rightarrow (\sqrt{3}x-\sqrt{2})=0$ or $(\sqrt{3}x-\sqrt{2})=0$

$\Rightarrow x=\sqrt{\frac{2}{3}},\ \sqrt{\frac{2}{3}}$

Solution 1(iii)

Given: $3\sqrt{2}x^2-5x-\sqrt{26}=0$

$\Rightarrow 3\sqrt{2}x^2-6x+x-\sqrt{2}=0$

$\Rightarrow 3\sqrt{2}x(x-\sqrt{2})+(x-\sqrt{2})=0$

$\Rightarrow (3\sqrt{2}x+1)(x-\sqrt{2})=0$

$\Rightarrow x=-\frac{1}{3\sqrt{2}}$ or $x=\sqrt{2}$

Question 2.
One root of the quadratic equation $8x^2 + mx + 15$ is 3/4. Find the value of m. Also, find the other root of the equation.

Solution:
Given quadratic equation is $8x^2 + mx + 15 = 0$ .... (i)
One of the roots of (i) is $\frac{3}{4}$, so it satisfies (i)

$$\Rightarrow 8\left(\frac{3}{4}\right)^2 + m\left(\frac{3}{4}\right) + 15 = 0$$

$$\Rightarrow \frac{9}{2} + 15 + m\left(\frac{3}{4}\right) = 0$$

$$\Rightarrow m\left(\frac{3}{4}\right) = -\frac{39}{2}$$

$$\Rightarrow m = -26$$

So, the equation (i) becomes $8x^2 - 26x + 15 = 0$
$\Rightarrow 8x^2 - 20x - 6x + 15 = 0$
$\Rightarrow 4x(2x - 5) - 3(2x - 5) = 0$
$\Rightarrow (4x - 3)(2x - 5) = 0$
$\Rightarrow x = \frac{3}{4}$ or $x = \frac{5}{2}$
$\Rightarrow x = \frac{3}{4}, \frac{5}{2}$
Hence, the other root is $\frac{5}{2}$

Question 3.
One root of the quadratic equation $x^2 - 3x - 2ax - 6a = 0$ is -3, find its other root.

Solution:
Given quadratic equation is .... (i)
One of the roots of (i) is -3, so it satisfies (i)
$\Rightarrow x^2 - 3x - 2ax - 6a = 0$
$\Rightarrow x(x + 3) - 2a(x + 3) = 0$
$\Rightarrow (x - 2a)(x + 3) = 0$
$\Rightarrow x = -3, 2a$
Hence, the other root is 2a.

Question 4.
If $p - 15 = 0$ and $2x^2 + 15x + 15 = 0$;find the values of x.

Solution:
Given i.e $p - 15 = 0$ i.e. $p = 15$
So, the given quadratic equation becomes
$2x^2 + 15x + 15 = 0$

⇒ 2x + 10x + 5x + 15 = 0
⇒ 2x(x + 5) + 5(x + 5)
⇒ (2x + 5)(x + 5) = 0
⇒ x = -5, $-\frac{5}{2}$
Hence, the values of x are -5 and $-\frac{5}{2}$

Question 5.
Find the solution of the equation $2x^2$ -mx − 25n = 0; if m + 5 = 0 and n − 1 = 0.

Solution:
Given quadratic equation is $2x^2$ -mx − 25n = 0 ….. (i)
Also, given and m + 5 = 0 and n − 1 = 0
⇒ m = -5 and n = 1
So, the equation (i) becomes
$2x^2$ + 5x + 25 = 0
⇒ 2x + 10x − 5x − 25 = 0
⇒ 2x(x + 5) -5(x + 5) = 0
⇒ (x + 5)(2x − 5) = 0
⇒ x = -5, $\frac{5}{2}$
Hence, the solution of given quadratic equation are x and $\frac{5}{2}$

Question 6.
If m and n are roots of the equation $\frac{1}{x} - \frac{1}{x-2} = 3$ where x ≠ 0 and x ≠ 2; find m × n.

Solution:

Given quadratic equation is $\frac{1}{x} - \frac{1}{x-2} = 3$
$\Rightarrow x - 2 - x = 3x(x - 2)$
$\Rightarrow -2 = 3x^2 - 6x$
$\Rightarrow 3x^2 - 6x + 2 = 0$
$\Rightarrow x = \frac{6 \pm \sqrt{6^2 - 4(3)(2)}}{2 \times 3}$
$\Rightarrow x = \frac{6 \pm \sqrt{12}}{2 \times 3}$
$\Rightarrow x = \frac{\sqrt{3} \pm 1}{\sqrt{3}}$

Since, m and n are roots of the equation, we have

$$\Rightarrow m = \frac{\sqrt{3}+1}{\sqrt{3}} \text{ and } n = \frac{\sqrt{3}-1}{\sqrt{3}}$$

$$\Rightarrow m \times n = \left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = \frac{2}{3}$$

$$m \times n = \frac{2}{3}$$

Question 7.
Solve, using formula :
$x^2 + x - (a + 2)(a + 1) = 0$

Solution:

Given quadratic equation is $x^2 + x - (a + 2)(a + 1) = 0$
Using quadratic formula,

$$\Rightarrow x = \frac{-1 \pm \sqrt{1^2 + 4(a+2)(a+1)}}{2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1 + 4(a^2 + 3a + 2)}}{2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{4a^2 + 12a + 9}}{2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{(2a+3)^2}}{2}$$

$$\Rightarrow x = \frac{-1 \pm (2a+3)}{2}$$

$$\Rightarrow x = \frac{-1 + (2a+3)}{2} \text{ or } x = \frac{-1 - (2a+3)}{2}$$

$$\Rightarrow x = \frac{2a+2}{2} \text{ or } x = \frac{-2a-4}{2}$$

$$\Rightarrow x = \frac{2(a+1)}{2} \text{ or } x = \frac{2(-a-2)}{2}$$

$$\Rightarrow x = a + 1 \text{ or } x = -a - 2 = -(a + 2)$$

Question 8.
Solve the quadratic equation $8x^2 - 14x + 3 = 0$
(i) When $x \in I$ (integers)
(ii) When $x \in Q$ (rational numbers)

Solution:
Given quadratic equation is $8x^2 - 14x + 3 = 0$
$\Rightarrow 8x^2 - 12x - 2x + 3 = 0$
$\Rightarrow 4x(2x - 3) - (2x - 3) = 0$
$\Rightarrow (4x - 1)(2x - 3) = 0$
$\Rightarrow x = \frac{3}{2}$ or $x = \frac{1}{4}$

(i) When $x \in I$, the equation $8x^2 - 14x + 3 = 0$ has no roots
(ii) When $x \in Q$ the roots of $8x^2 - 14x + 3 = 0$ are
$x = \frac{3}{2}$ $x = \frac{1}{4}$

Question 9.
Find the value of m for which the equation $(m + 4)^2 + (m + 1)x + 1 = 0$ has real and equal roots.

Solution:
Given quadratic equation is $(m + 4)^2 + (m + 1)x + 1 = 0$
The quadratic equation has real and equal roots if its discriminant is zero.
$\Rightarrow D = b^2 - 4ac = 0$
$\Rightarrow (m + 1)^2 - 4(m + 4)(1) = 0$
$\Rightarrow m^2 + 2m + 1 - 4m - 16 = 0$
$\Rightarrow m^2 - 2m - 15 = 0$
$\Rightarrow m^2 - 5m + 3m - 15 = 0$
$\Rightarrow m(m - 5) + 3(m = 5) = 0$
$\Rightarrow (m - 5)(m + 3) = 0$
$\Rightarrow m = 5$ or $m = -3$

Question 10.
Find the values of m for which equation $3x^2 + mx + 2 = 0$ has equal roots. Also, find the roots of the given equation.

Solution:
Given quadratic equation is $3x^2 + mx + 2 = 0$ .... (i)
The quadratic equation has equal roots if its discriminant is zero
$\Rightarrow D = b^2 - 4ac = 0$
$\Rightarrow m^2 - 4(2)(3) = 0$
$\Rightarrow m^2 = 24$
$\Rightarrow m = \pm 2\sqrt{6}$
When $m = 2\sqrt{6}$, equation (i) becomes

$$3x^2 + 2\sqrt{6}x + 2 = 0$$
$$\Rightarrow \left(\sqrt{3}x + \sqrt{2}\right)^2 = 0$$
$$\Rightarrow x = -\frac{\sqrt{2}}{\sqrt{3}} = -\frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{6}}{3}$$

When m = $-2\sqrt{6}$, equation (i) becomes

$$3x^2 - 2\sqrt{6}x + 2 = 0$$
$$\Rightarrow \left(\sqrt{3}x - \sqrt{2}\right)^2 = 0$$
$$\Rightarrow x = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

$\therefore x = -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}$

Question 11.
Find the value of k for which equation $4x^2 + 8x - k = 0$ has real roots.

Solution:
Given quadratic equation is $4x^2 + 8x - k = 0$ .... (i)
The quadratic equation has real roots if its discriminant is greater than or equal to zero
$\Rightarrow D = b^2 - 4ac \geq 0$
$\Rightarrow 8^2 - 4(4)(-k) \geq 0$
$\Rightarrow 64 + 16k \geq 0$
$\Rightarrow 16k \geq -64$
$\Rightarrow k \geq -4$
Hence, the given quadratic equation has real roots for $k \geq -4$

Question 12.
Find, using quadratic formula, the roots of the following quadratic equations, if they exist
(i) $3x^2 - 5x + 2 = 0$
(ii) $x^2 + 4x + 5 = 0$

Solution:
(i) Given quadratic equation is $3x^2 - 5x + 2 = 0$
$D = b^2 - 4ac = (-5)^2 - 4(3)(2) = 25 - 24 = 1$
Since $D > 0$, the roots of the given quadratic equation are real and distinct.
Using quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(2)}}{2(3)}$$

$$\Rightarrow x = \frac{5 \pm \sqrt{25 - 24}}{6}$$

$$\Rightarrow x = \frac{5 \pm 1}{6}$$

$$\Rightarrow x = \frac{5+1}{6} \text{ or } x = \frac{5-1}{6}$$

$$\Rightarrow x = \frac{6}{6} \text{ or } x = \frac{4}{6}$$

$\Rightarrow x = 1$ or $x = \frac{2}{3}$

(ii) Given quadratic equation is $x^2 + 4x + 5 = 0$
$D = b^2 - 4ac = (4)^2 - 4(1)(5) = 16 - 20 = -4$

Since $D < 0$, the roots of the given quadratic equation does not exist.

Solution 13:

(i) Given quadratic equation is $\frac{1}{18-x} - \frac{1}{18+x} = \frac{1}{24}$

$$\Rightarrow \frac{(18+x)-(18-x)}{(18+x)(18-x)} = \frac{1}{24}$$

$$\Rightarrow \frac{2x}{18^2 - x^2} = \frac{1}{24}$$

$\Rightarrow 48x = 324 - x^2$
$\Rightarrow x^2 + 48x - 324 = 0$
$\Rightarrow x^2 + 54x - 6x - 324 = 0$
$\Rightarrow x(x + 54) - 6(x + 54) = 0$
$\Rightarrow (x + 54)(x - 6) = 0$
$\Rightarrow x = -54$ or $x = 6$
But as $x > 0$, so x can't be negative.
Hence, $x = 6$.

(ii) Given quadratic equation is $(x - 10)\left(\frac{1200}{x} + 2\right) = 1260$
$\Rightarrow (x - 10)^{\left(\frac{1200+2x}{x}\right)} = 1260$

$\Rightarrow (x - 10)(1200 + 2x) = 1260x$
$\Rightarrow 1200x + 2x^2 - 12000 - 20x = 1260x$
$\Rightarrow 2x^2 - 12000 - 80x = 0$
$\Rightarrow x^2 - 40x - 6000 = 0$
$\Rightarrow x^2 - 100x + 60x - 6000 = 0$
$\Rightarrow (x - 100)(x - 60) = 0$
$\Rightarrow x = 100$ or $x = -60$
But as $x < 0$, so x can't be positive.
Hence, $x = -60$.